

Measuring and Controlling Shortfall Risk in Retirement

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Abstract

A key challenge for retired investors is determining the stock-bond asset allocation that, for a given spending rate, provides an acceptable probability of shortfall—having real wealth drop below a specified floor during the investor’s lifetime. Standard portfolio analysis yields the well-known tradeoff between risk and return described by the Markowitz frontier. For retirement planning, we reconceptualize this as a tradeoff between shortfall probability (risk) and the median value of terminal wealth (return). For specified assumptions, there is a stock-bond asset allocation that minimizes shortfall risk. Portfolios with more stocks increase the median values of terminal wealth, but at the expense of higher shortfall risk. Portfolios with less stocks are inferior in that they decrease the median value of terminal wealth *and* increase shortfall risk. We find that for a variety of plausible assumptions about asset returns, investment strategies, and what constitutes a shortfall, the minimum-risk portfolio generally has between 50 and 70 percent stocks.

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Bobby Layne, a hall-of-fame football quarterback, once said, “My idea of a full life is to run out of money and breath at the same time.” This pithy quip dramatizes the reality that the elderly want to enjoy their retirement years, but fear that they may exhaust their savings prematurely. Whether or not they will outlive their wealth depends on two crucial decisions: (1) how much of their wealth they spend each year (their withdrawal rate); and (2) how they allocate their wealth among risky assets with uncertain rates of return. This paper uses a Monte Carlo simulation model to show how retirees might assess the tradeoffs among their standard of living, the chances that their wealth will turn out to be inadequate, and the size of their bequest.

Standard portfolio analysis yields a well-known tradeoff between risk and return described by the Markowitz frontier. For retirement planning, we reconceptualize this as a tradeoff, for a given spending rate, between shortfall probability and the median value of terminal wealth. For specified assumptions, there is a stock-bond asset allocation that minimizes shortfall risk. Portfolios with more stocks increase the median values of terminal wealth, but at the expense of higher shortfall risk. Portfolios with less stocks are inferior in that they decrease the median value of terminal wealth *and* increase shortfall risk. We illustrate this approach with a variety of plausible assumptions about asset returns and spending and investment strategies.

The Model

The evolution of a household’s wealth over time depends on a large number of socioeconomic factors and also on uncertain asset returns and mortality. Our baseline case considers a household with simplified demographic and economic characteristics. Because the theoretical probability distribution of the evolution of wealth is intractable, we use a Monte Carlo simulation model to estimate this probability distribution.

Demographics

Most retirement planning models assume a single person (for example, Bengen 1994; Milevsky and Robinson 2000; Shoven 1999; Booth 2004). Our model can handle this scenario, but a two-person household is more realistic. We consider a couple, each of whom was born in 1940; their retirement plan starts in 2005, when each is 65 years old.

Using life tables prepared by the Office of the Chief Actuary in the Social Security Administration, the Berkeley Mortality Database (2003) provides historical death rates for 1900-1995 and projected death rates for 1996-2080. These data show either the historical frequency or estimated probability of death each year for males and females of ages 0 to 120. We assume that our couple are an average male (with an implied life expectancy of 17 years, to age 82) and an average female (with an implied life expectancy of 21 years, to age 86).

Some authors (for example, Bengen 1994, Pye 2000; Booth 2004) calculate the probability that wealth will be exhausted over specified horizons, which can be compared to a retiree's life expectancy. However, there is only a small probability that a person's life will exactly match his or her life expectancy. Just as it is better to use a probability distribution for an asset's annual return than to assume that the annual return will equal its historical average, so it is better to use annual mortality probabilities than to assume that everyone with a 20-year life expectancy lives exactly 20 years. Matters are further complicated by the fact that if a husband and wife each have a life expectancy of 20 years, the joint life expectancy is greater than 20 years.

We consequently use the Berkeley Mortality Database to determine each person's mortality probability in each year of the simulation and a random number generator to determine whether this person dies or lives another year.

Financial Resources

Our hypothetical couple has \$1 million in financial assets and no income other than what is

generated by their assets. Because we specify the withdrawal rate as a fraction of their initial wealth, the results are scalable. That is, the shortfall probability will be the same for a couple with \$2 million in initial assets that spends twice as much as our hypothetical couple. Social Security benefits could be introduced by assuming that the couple spends all of its Social Security benefits in addition to the withdrawals from its assets.

We ignore housing, thereby implicitly assuming that the couple doesn't foresee any housing transactions that will affect its financial wealth. Venti and Wise (2001) find that retirees generally do not liquidate their housing equity to finance spending. We also follow the common practice of ignoring taxes (Bengen 1994; Milevsky 1997; Cooley, Hubbard, and Walz 1998).

Withdrawal Rate

Many people would like to maintain a stable lifestyle, while spending as much as they prudently can without a substantial risk of their wealth falling sufficiently to endanger their spending or their target bequest. Retirement planning models generally assume that annual spending is initially equal to a specified fraction of initial wealth (the "withdrawal rate") and that spending is then adjusted each year for inflation (Bengen 1994; Warshawsky 2000; Milevsky, 2001, 2005). Bengen and Warshawsky recommend a 4% inflation-adjusted withdrawal rate; Milevsky (2001) suggests 7.5% for risk-tolerant retirees (who are willing to accept a 10-to-25 percent chance of outliving their money); Milevsky (2005) recommends 4.5% to 5%. Our baseline case considers three scenarios: 3%, 4%, or 5% withdrawal rates, which correspond to annual real spending of \$30,000, \$40,000, or \$50,000.

The authors asked seven certified financial planners and insurance agents how much they expected spending by a retired household to decline when either spouse dies. Most said that for retired couples, neither of whom had extraordinary medical costs, they assume a 20%-25% drop in spending. Our baseline case assumes a 25% decline; we also consider 0% and 50% declines as

boundaries.

Investing

Retirement planning models (for example, Bengen 1994, Kim and Wong 1997, Shoven 2000) typically consider two assets—corporate stock and Treasury bonds—and we will too, with the percentage of wealth invested in stocks varying from 0 to 100. The portfolio is rebalanced annually to achieve the desired stock-bond allocation.

In principle, investors should use asset-return distributions that reflect their personal beliefs. For example, many observers, including Campbell, Diamond, and Shoven (2001); Carlson, Pelz & Wohar (2002); Ilmanen (2003); and Siegel (1992), argue that, in comparison to the returns experienced in the twentieth century, future average stock returns are likely to be lower and closer to the average return on Treasury bonds—a projection often described as the shrinking equity premium.

In practice, investment simulation models usually assume that future returns are randomly drawn from either (a) theoretical probability distributions with parameters estimated from historical returns; or (b) the actual distribution of historical returns (for example, Canner, Mankiw, and Weil 1997; Cooley, Hubbard, and Walz 1998; Pye 2000; Hickman, *et al* 2001; Marbach 2002). For illustrative purposes, our baseline case assumes that annual real stock and bond returns are drawn from lognormal distributions with the historical means, standard deviations, and correlation coefficient. The results using draws from the annual historical data are similar and are available from the authors.

Shortfall

One way to measure the riskiness of an investment strategy is by the standard deviation of terminal wealth. However, investors typically choose portfolios as if short-term (for example, annual) fluctuations in market value are important and as if they are more concerned about losses

than gains—what Benartzi and Thaler (1995) call “myopic loss aversion.” One way of handling the asymmetric valuation of gains and losses is with the semivariance (Markowitz 1959) or, equivalently, the downside deviation (Sortino and Price, 1994), which is the square root of the semi-variance.

Another way to gauge outcomes is by percentiles of the terminal wealth distribution; for example, the 5th, 50th, and 95th percentiles (Scott 2002). Other authors (Milevsky and Robinson 2000) measure outcomes by the probability that wealth will be zero before death—a person outlives her wealth. Zero wealth is a retiree’s worst-case scenario; they may also be unsettled if their wealth falls to a level that threatens to plunge them into their worst-case scenario or jeopardizes their bequest intentions. Roy (1952) argues that investors think in terms of a minimum acceptable outcome, what he calls the “disaster level.” The safety-first strategy is to choose the investment with the smallest probability of going below the disaster level. We define a shortfall as an outcome in which real wealth in any year is 50% less than initial wealth; we also look at 75% and 100% shortfalls. We assume that investors use the shortfall probability as a measure of risk, but do not necessarily adopt the safety-first strategy by choosing the investment strategy with the smallest shortfall probability.

Results

For n independent simulations, each with a shortfall probability p , the simulation standard error for the sample success proportion is approximately

$$\sqrt{\frac{p(1-p)}{n}}$$

One million simulations were used for each of our scenarios, with a maximum standard error of the shortfall probability equal to 0.0005.

For our initial simulations, the annual real stock and bond returns are generated by lognormal

probability distributions with means, standard deviations, and correlation coefficient estimated from the historical data for 1926-2004 compiled by Ibbotson and Sinquefeld (2005). Specifically, stocks have a mean of 9.2% and a standard deviation of 20.4%; bonds have a mean of 2.8% and a standard deviation of 10.4%; the correlation between stock and bond returns is 0.20.

Exhibits 1 and 2 show the tradeoff between shortfall probability and median value of real terminal wealth for withdrawal rates of 3%, 4%, and 5%. (The 11 points on the lines in Exhibit 2 show stock allocations ranging from 0% to 100% in 10% increments.) The shortfall probability on the horizontal axis is the probability that the household will experience a 50% decline in real (inflation-adjusted) wealth sometime during their lives; the median value of terminal wealth on the vertical axis is the median real bequest after both persons have died.

In mean-variance analysis, the Markowitz frontier describes the tradeoff between the expected value and standard deviation of the portfolio return. Analogously, the data in Exhibits 1 and 2 describe the tradeoff between shortfall probability (risk) and median terminal wealth (return). As with the Markowitz frontier, the leftmost point on the curve is the minimum-risk position and opportunities below this point are dominated by this minimum-risk position since, for any given shortfall probability, households prefer higher median terminal wealth. For a 4% withdrawal rate, the minimum shortfall probability (Roy's safety-first portfolio) is with a 57% stock percentage; portfolios with less than a 57% stock percentage are dominated by portfolios with larger stock percentages. The minimum-risk portfolio is 48% stocks with a 3% withdrawal and 69% stocks with a 5% withdrawal.

An increasing stock percentage always increases the expected value and variance of terminal wealth; however, starting from a 0% stock position, an increasing stock percentage initially reduces the shortfall probability. If shortfall probability is the relevant measure of risk, then the inclusion of modest amounts of stock in a retiree's portfolio actually reduces risk even though the

variance of the portfolio return is increasing. This is because the increase in the portfolio's expected return dominates the increased variability of the return. Once the equity percentage passes the leftmost point on the curve (57% stocks for a 4% withdrawal rate), the increased variability dominates the increased expected return and the shortfall risk begins increasing. Beyond the minimum-risk portfolio, the household's preferred stock-bond allocation depends on its risk preferences.

Using a 4% withdrawal rate, Exhibits 3 and 4 show how increasing the shortfall threshold from 50% to 75% to 100% shifts the tradeoff leftward. The minimum shortfall probability is 0.13 (with 57% stocks) when a shortfall is defined as a 50% drop in wealth, 0.05 (with 56% stocks) when a shortfall is defined as a 75% drop in wealth, and 0.02 (with 52% stocks) when a shortfall is defined as a 100% drop in wealth.

Exhibit 5 compares the tradeoffs for a 0% or 50% drop in spending after the death of a spouse to the 25% baseline value. As expected, the shortfall probabilities are higher the smaller the drop in spending. Here the minimum shortfall probabilities are 0.11 (with 54% stocks) for a 50% drop, 0.13 (with 57% stocks) for a 25% drop, and 0.15 (with 62% stocks) for no drop.

Overlapping and Nonoverlapping Sequences

One of the biggest risks investors face is not an unlucky draw from a stable probability distribution, but a substantial change in the probability distribution itself. Suppose, for example, that an investor assumes that annual real stock returns are well described as random draws from a normal probability distribution with a mean of 10% and a standard deviation of 20%. If so, the probability of a loss of more than 30% in any year is 0.02. But now suppose the economic environment changes so that, for an extended period of time, real stock returns are random draws from a normal probability distribution with a mean of 5% and a standard deviation of 30%. The probability of a loss of more than 30% rises to 0.12.

We illustrate this situation in two ways: by shrinking the equity premium permanently and by allowing the probability distribution to change every 15 years. For the first approach, we assume that annual real stock and bond returns are drawn from lognormal distributions with the historical means, standard deviations, and correlation coefficients with one difference—the mean stock return is reduced by 3 percentage points, thereby reducing the equity premium by 3 percentage points. Exhibit 6 compares the tradeoff using this reduced equity premium with the tradeoff using the unadjusted historical data. The 3-percentage-point drop in the equity premium drastically reduces the median value of terminal wealth and increases the shortfall probability for portfolios with substantial stock holdings. The minimum-risk portfolio changes from 57% stocks to 45% stocks.

For our second illustration of a changed probability distribution, we randomly separated the historical data into nonoverlapping 15-year sequences. One randomly selected sequence is used to estimate the mean returns for stocks and bonds; a second sequence is used to estimate the standard deviations and correlation coefficient. These parameters remain in effect for 15 years in that, for 15 years of the simulation the annual returns are drawn from lognormal distributions with these parameters. After 15 years, new 15-year sequences are selected to yield new values for the means, standard deviations, and correlation coefficient. Exhibit 7 compares the tradeoff for this scenario with that for the original scenario of annual draws from stable lognormal distributions with means, standard deviations, and correlation coefficient estimated from the complete set of historical data. As shown, the tradeoff is substantially less favorable when the means and standard deviations may change abruptly. But, again, there is relatively little change in the minimum-risk portfolio, from 57% stocks to 65% stocks.

An appealing way to implement this idea of changing probability distributions is with a hierarchical model that specifies a joint probability distribution for the means, standard

deviations, and correlation coefficient. Every n years, new values of the model's parameters are chosen randomly. We will pursue that approach in future research.

Your Age in Bonds

Samuelson (1963, 1969) and Merton (1969) show that a rational risk-averse investor's optimal bond-stock allocation for a fixed horizon of length T does not depend on the value of T . Samuelson (1994) writes that "it is an exact theorem that investment horizons have no effect on your portfolio proportions." Nonetheless, Kritzman (1994) offers several reasons why an investor's risk exposure might depend on the time horizon and, in practice, many financial advisors recommend holding more bonds as they grow older [O'Connell 1995; Greninger, *et al* 2000; Malkiel 1990]. For example, life cycle funds such as Vanguard's Target Retirement Funds grow more conservative as the investor ages by increasing the bond allocation and reducing the stock allocation. Many financial planners use the "your age in bonds" rule; that is 50% bonds at age 50, 60% at age 60, and so on (Canner, Mankiw, and Weil 1997; Booth 2004).

Booth argues that the bond proportion should increase as retirement age approaches. However, his model does not track an individual's decisions over time. Instead, he compares different investors with different horizons, each of whom needs the same target return to meet their goals. Thus, he argues that a person with a one-year horizon who can meet his goals with safe bonds should do so, while a person with a longer horizon who can only meet their goals by including stocks in his portfolio should do that.

Jagannathan and Kocherlakota (1996) investigate three common economic reasons for increasing bond percentages over the life cycle. They find that the only justifiable motive is as part of a portfolio strategy involving stocks, bonds, and future labor income; as people age and future labor income declines, it may be optimal to increase bonds relative to stocks. However, Heaton and Lucas (1997) argue that observed volatility and correlations do not justify the

magnitude of observed bond holdings; this apparent underinvestment in stocks is part of a continuing equity-premium mystery. In any case, we are dealing with retirees, for whom changes in future labor income are not an issue and therefore should not affect the optimal bond-stock ratio.

Exhibit 8 shows that the age-in-bonds strategy is slightly to the left of the fixed-percentage tradeoff, but is nonetheless an inefficient strategy since it is southeast of the minimum-risk portfolio. Looking horizontally, the age-in-bonds strategy has a slightly lower shortfall probability than does the corresponding fixed allocation with the same median bequest. However, the age-in-bonds strategy is dominated by other fixed allocations. For example, the age-in-bonds strategy has a 0.200 shortfall probability and a \$1.06 million median bequest, but a 57% stock portfolio has a 0.128 shortfall probability and \$1.71 million median bequest.

The fundamental problem with the age-in-bonds strategy for a retired person is that the average bond percentage is inefficiently high, in that more stocks in the portfolio would increase the median bequest while reducing the shortfall probability. The age-in-bonds rule commits anyone past the age of 57 to portfolios that, historically, have been dominated by portfolios with more stocks and fewer bonds.

One way to unleash the age-in-bonds strategy is to modify it to an age-minus-25 or age-minus-35 strategy. With an age-minus-25 strategy, the couple that retires at age 65 begins with a 40% bond portfolio; at age 85, the portfolio is 60% bonds. Exhibit 8 shows that the age-minus-25 strategy is still inefficient in that it is (slightly) southwest of the 57%-stock portfolio. The age-minus-35 strategy does better, but (with a 0.130 shortfall probability and \$1.775 million median bequest) is essentially equivalent to a fixed 60%-stock portfolio (with a 0.128 shortfall probability and \$1.779 million median bequest).

Flexible Spending

Although, *ex ante*, they prefer a smooth consumption path, retirees *ex post* may want to adjust their spending as their wealth fluctuates [Ameriks, Caplin, and Leahy 2003]. We model this by allowing our hypothetical couple to have a wealth elasticity of spending of 50%; for example, if their real wealth changes by 10% relative to initial wealth, their real spending changes by 5% relative to initial spending. In our baseline case, wealth is initially \$1,000,000 and spending is \$40,000. If real wealth rises to \$1,100,000 or falls to \$900,000, then real spending changes to \$42,000 or \$38,000, respectively. Exhibits 9 and 10 show the tradeoffs in comparison to fixed real spending equal to 4% of initial wealth. The minimum-risk portfolio is 51% stocks.

The flexible-spending strategy dominates the fixed 4% withdrawal rate in that, for any given stock-bond allocation, the flexible spending strategy with the same stock-bond allocation has a smaller shortfall probability and there is generally a flexible-spending strategy with a different stock-bond allocation that has a smaller shortfall probability and higher median bequest. For example, a fixed 4% withdrawal with a 60% stock portfolio has a shortfall probability of 0.128 and a median bequest of \$1.78 million, while a 50% spending elasticity with a 60% stock portfolio has a shortfall probability of 0.098 and a median bequest of \$1.69 million. A 50% spending elasticity with a 70% stock portfolio has a shortfall probability of 0.107 and a median bequest of \$1.87 million. The price of this reduced shortfall probability and increased median bequest is lower consumption when wealth declines.

Another flexible-spending strategy is to adjust spending as wealth fluctuates, but never let the real value of spending fall below its initial level. In our baseline case, if real wealth rises from its initial \$1,000,000 level to \$1,100,000, real spending increases from its initial \$40,000 level to \$42,000; if real wealth falls to \$900,000, real spending stays at the \$40,000 floor. This behavior is consistent with the choice of a minimum real spending level that has an acceptable shortfall probability. Exhibits 9 and 10 show these tradeoffs. Interestingly, the minimum-risk portfolio is

57% stocks, the same as in the baseline case.

Exhibit 9 shows that for any given stock-bond allocation, flexible spending without a floor has the lowest shortfall probability because spending is curtailed if wealth begins falling. If the household does not want real spending to fall below its initial level, then flexible spending with a floor is an attractive alternative to a fixed spending level in that spending is allowed to increase when wealth does, but there is only a slight rise in the shortfall probability (because of the possibility that wealth will increase and then decline: if spending increases with wealth, there is less wealth to cushion the subsequent decline). However, the median bequest is lower relative to both of the other strategies because spending increases when wealth does and spending is not curtailed when wealth falls below its initial level.

Discussion

A classic paper by Bengen (1994) looks at retirement portfolios started in different historical time periods that realize the actual sequence of historical stock and bond returns over the succeeding years. He concludes that a 4% withdrawal rate is safe in that “in no past case has it caused a portfolio to be exhausted before 33 years.” In looking at different stock-bond allocations, Bengen concludes that “the 50/50 stock/bond mix appears to be near-optimum for generating the highest minimum portfolio longevity for any withdrawal scheme.” However because a 75-25 allocation would have generally increased terminal wealth, Bengen recommends “a stock allocation as close to 75 percent as possible, and in no cases less than 50 percent.”

The past can provide general guidance regarding plausible and implausible asset returns. However, it is extremely unlikely that the future will replicate the past exactly. If one has definite beliefs about future assets returns, it is sensible to work with an explicit probability distribution that accurately reflects these beliefs. If one is unwilling to specify an explicit probability distribution, then it is sensible to apply a sensitivity analysis by examining a variety of plausible

scenarios that are similar, but not necessarily identical, to the historical data.

For a given withdrawal rate, one can determine the tradeoff between shortfall probability and the median value of terminal wealth offered by various asset-allocation strategies. Using this framework, one can then choose a withdrawal rate and asset-allocation strategy that yields the preferred combination of shortfall probability and median bequest. We find that for a variety of plausible assumptions about asset returns, investment strategies, and what constitutes a shortfall, the minimum-risk portfolio generally has between 50 and 70 percent stocks. This is not to say that one should necessarily choose the minimum-risk portfolio. A household may choose a portfolio that is more heavily invested in stocks, thereby accepting an increased shortfall probability in return for higher median terminal wealth. What we can say is that portfolios that are less heavily invested in stocks are inefficient in that they are dominated by stock heavy portfolios that give a lower shortfall probability and higher median terminal wealth.

An age-in-bonds strategy is often recommended by financial advisers but, historically at least, such a strategy would have given retirees inefficiently bond-heavy portfolios, in that more stocks and less bonds would have reduced the shortfall probability and increased the median bequest. An age-minus-35 strategy corrects this problem, but is not an improvement over a fixed 60% stock portfolio.

A flexible-spending strategy that adjusts spending as wealth fluctuates is very appealing in that it can substantially reduce the shortfall probability and also increase the median bequest; the cost is reduced consumption when wealth declines. A variation is to increase spending if wealth rises above its initial level, but not reduce spending below its initial level if wealth falls below its initial level. This flexible-spending-with-a floor strategy allows spending to increase with wealth and has little effect on the shortfall probability.

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Exhibit 1 Shortfall Probabilities for 3%, 4%, and 5% Withdrawal Rates

Stock Percent	Withdrawal Rate		
	3%	4%	5%
100	0.126	0.180	0.245
90	0.106	0.161	0.232
80	0.089	0.146	0.221
70	0.075	0.134	0.217
60	0.066	0.128	0.221
50	0.061	0.130	0.236
40	0.064	0.144	0.268
30	0.077	0.177	0.323
20	0.110	0.237	0.404
10	0.172	0.327	0.504
0	0.270	0.439	0.604

A shortfall is defined as a 50% decline in real wealth

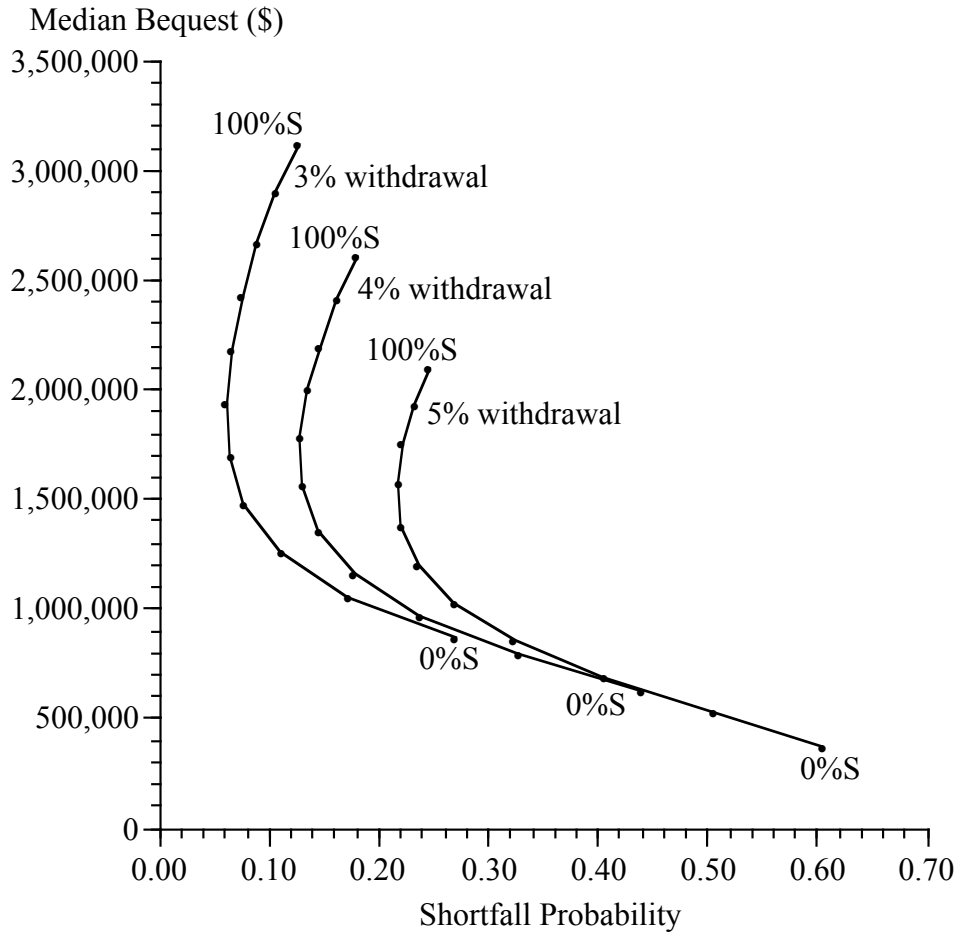


Exhibit 2 The tradeoff between shortfall probability and median terminal wealth

A shortfall is defined as a 50% decline in real wealth

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

Exhibit 3 Shortfall Probabilities for Different Shortfall Criteria, 4% Withdrawal Rate

Stock Percent	Shortfall Drop in Wealth		
	50%	75%	100%
100	0.180	0.075	0.036
90	0.161	0.064	0.029
80	0.146	0.056	0.024
70	0.134	0.050	0.020
60	0.128	0.047	0.018
50	0.130	0.047	0.017
40	0.144	0.053	0.018
30	0.177	0.068	0.023
20	0.237	0.099	0.035
10	0.327	0.155	0.059
0	0.439	0.238	0.101

A shortfall is defined as a 50% decline in real wealth

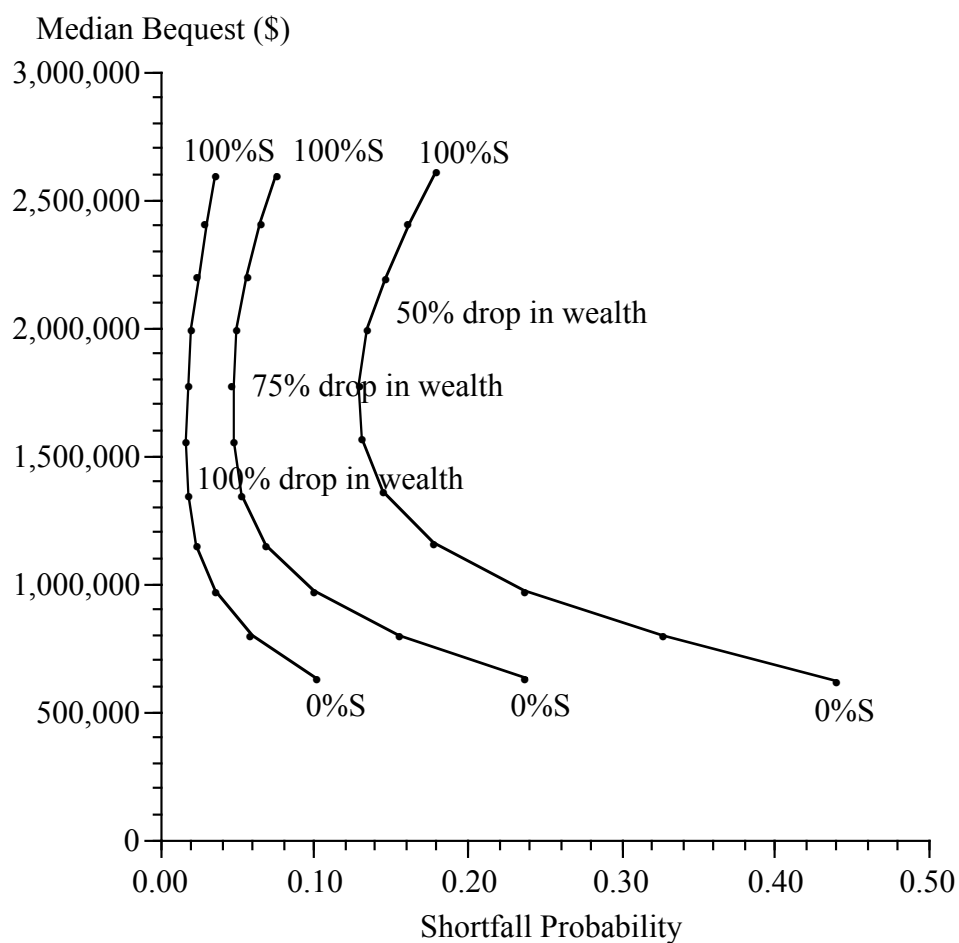


Exhibit 4 The tradeoff between shortfall probability and median terminal wealth for a 4% withdrawal rate and shortfalls defined as declines in real wealth of 50%, 75%, and 100%

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

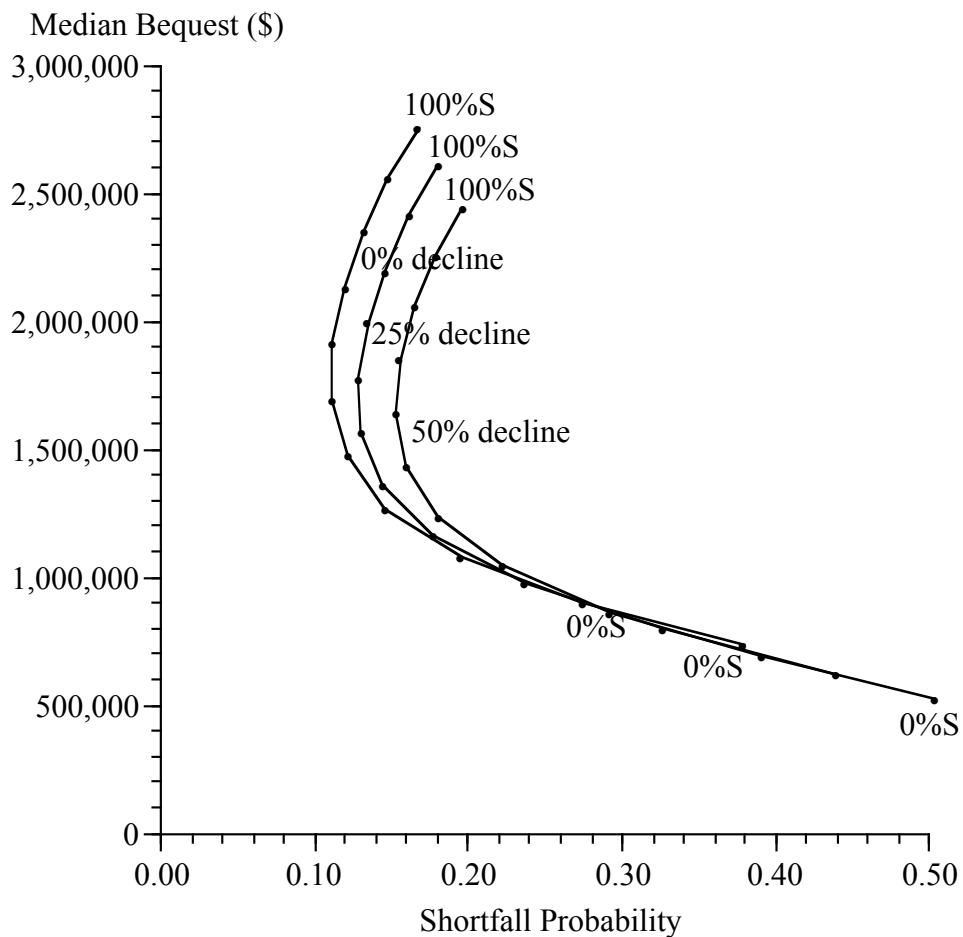


Exhibit 5 The tradeoff between shortfall probability and median terminal wealth for 0%, 25%, and 50% decline in spending when spouse dies

A shortfall is defined as a 50% decline in real wealth

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

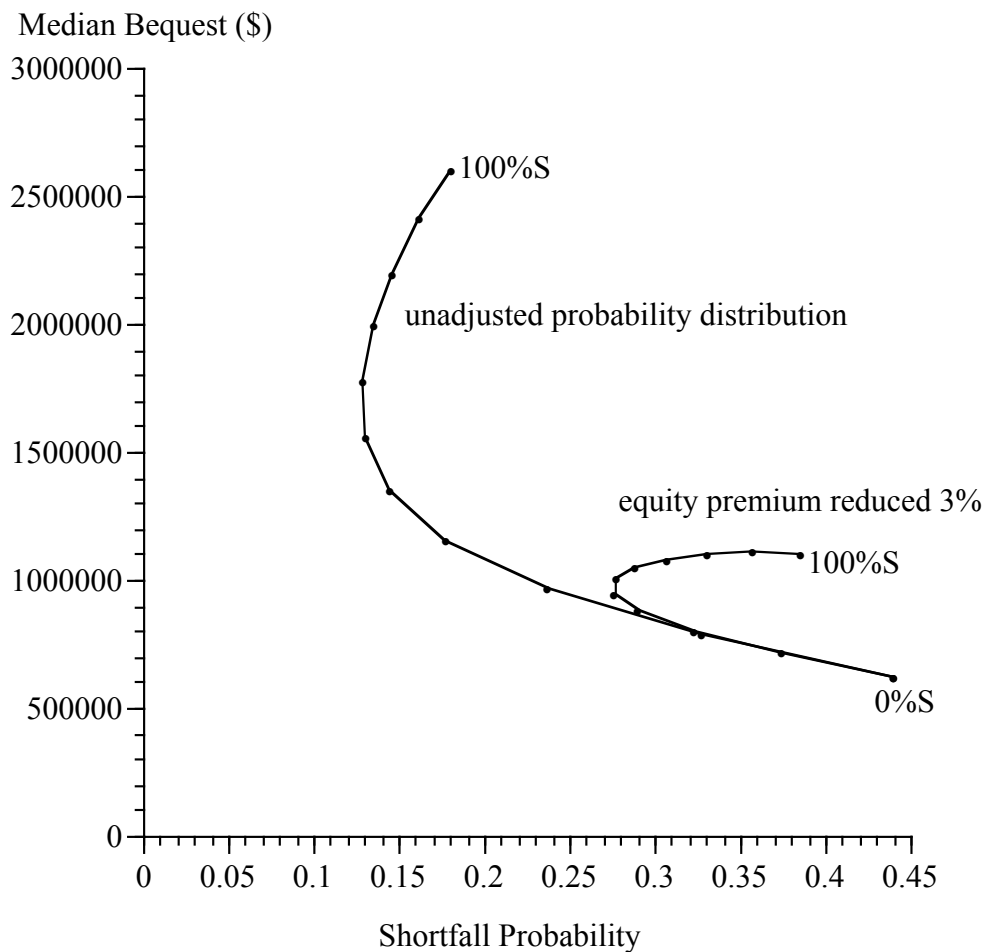


Exhibit 6 Unadjusted Probability Distribution and Equity Premium reduced by 3%, 4% withdrawal rate

A shortfall is defined as a 50% decline in real wealth

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

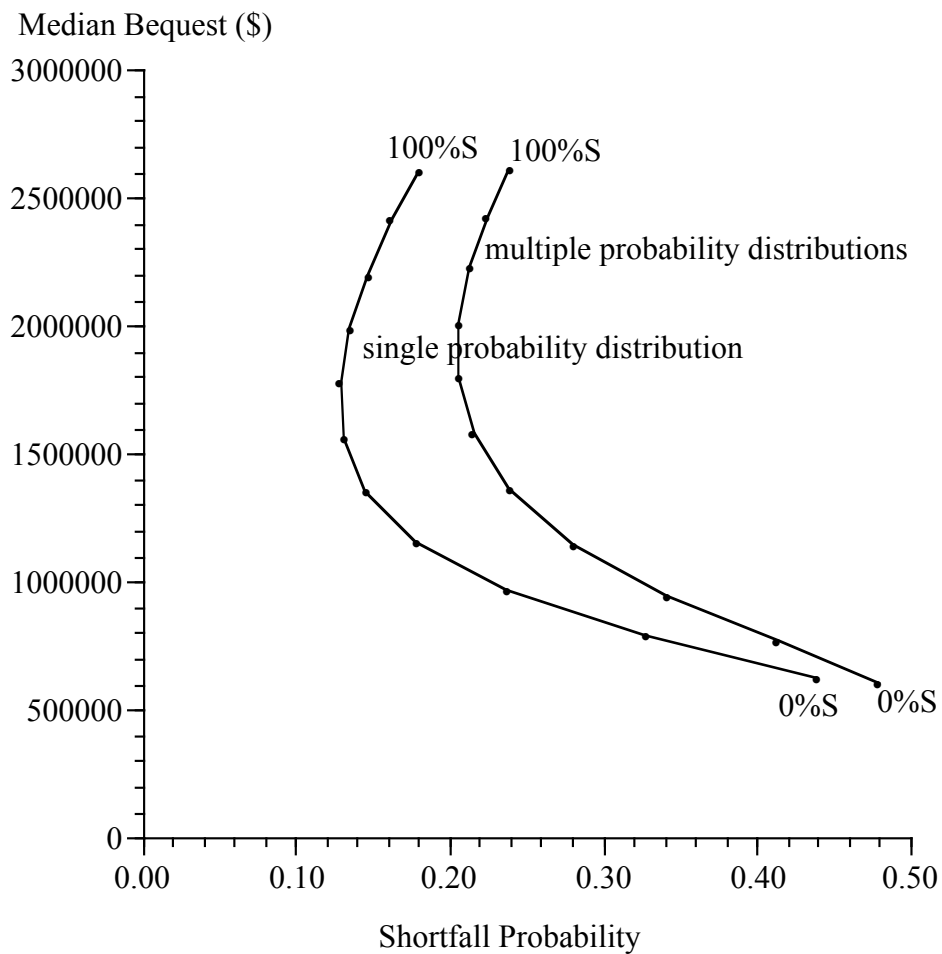


Exhibit 7 Single and Multiple Estimated Probability Distributions, 4% withdrawal rate

A shortfall is defined as a 50% decline in real wealth

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

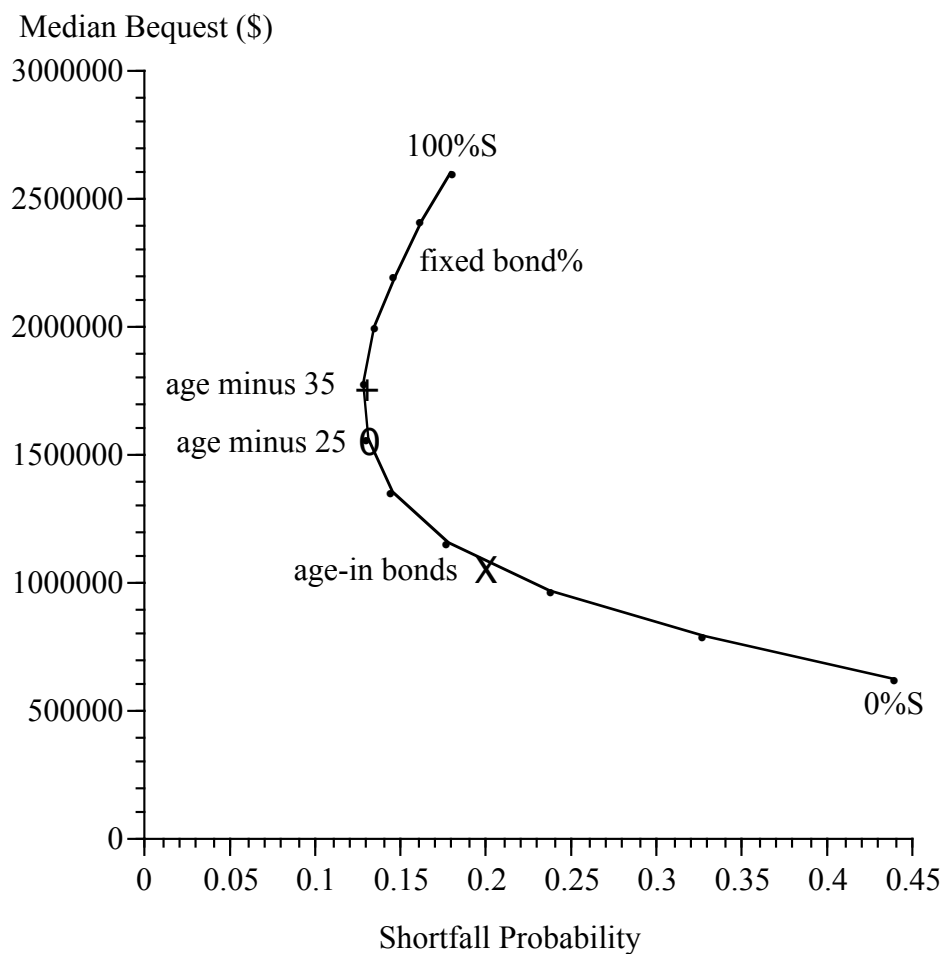


Exhibit 8 Three bond-age strategies compared to fixed bond percentages, 4% withdrawal rate

A shortfall is defined as a 50% decline in real wealth

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

X : age in bonds

0 : age-minus-25 in bonds

+: age-minus-35 in bonds

Each curve depicts 11 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments

Exhibit 9 Shortfall Probabilities for Different Wealth Elasticities

Stock Percent	Wealth Elasticity		
	0%	50%	50% with Floor
100	0.180	0.161	0.185
90	0.161	0.140	0.165
80	0.146	0.122	0.149
70	0.134	0.107	0.137
60	0.128	0.098	0.131
50	0.130	0.095	0.133
40	0.144	0.102	0.148
30	0.177	0.125	0.181
20	0.237	0.172	0.241
10	0.327	0.253	0.332
0	0.439	0.364	0.445

A shortfall is defined as a 50% decline in real wealth.

50% wealth elasticity: If real wealth changes by X%, real spending changes by 0.5X%

50% with floor: If real wealth is X% higher than the initial level, real spending is 0.5X% higher than its initial level

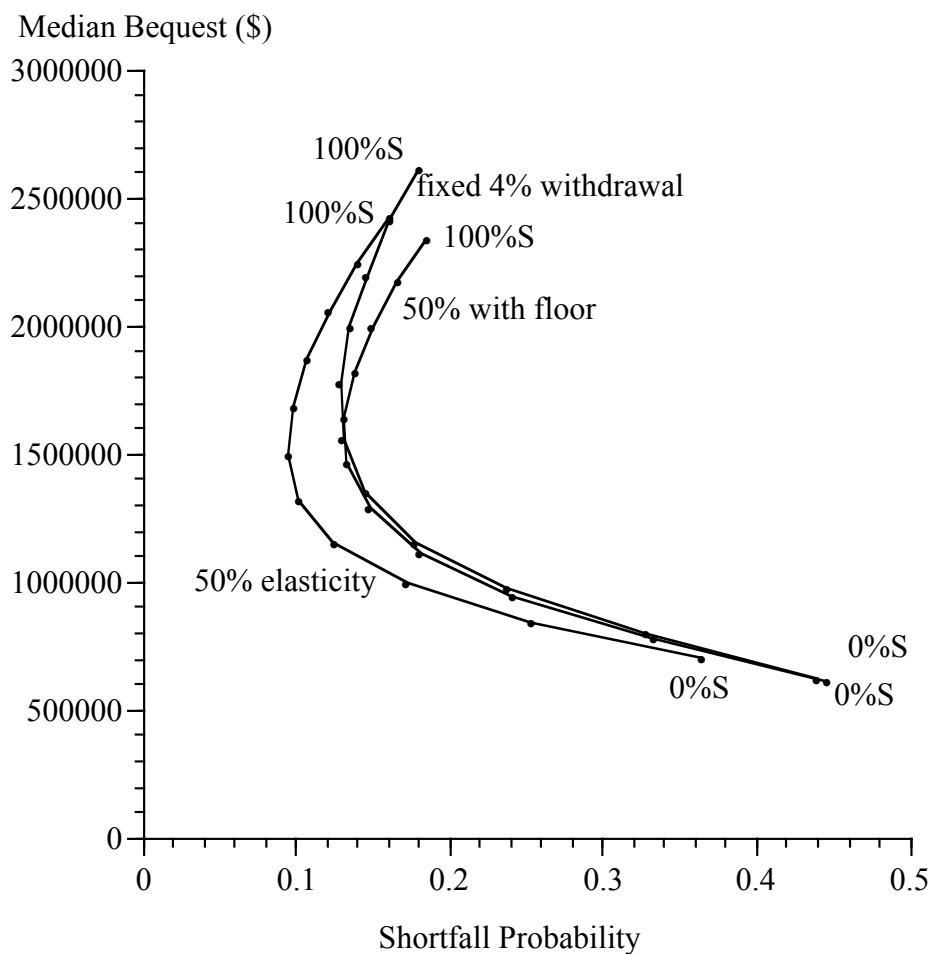


Exhibit 10 50% elasticity of spending with respect to wealth versus fixed 4% withdrawal rate

A shortfall is defined as a 50% decline in real wealth

50% wealth elasticity: If real wealth changes by X%, real spending changes by 0.5X%

50% with floor: If real wealth is X% higher than the initial level, real spending is 0.5X% higher than its initial level

0%S: 0% stocks, 100% bonds

100%S: 100% stocks, 0% bonds

Each curve depicts 10 portfolios with portfolio allocations ranging from 0% stocks to 100% stocks in 10% increments