SOLUTIONS TO CHAPTER 9

Problem 9.1

(a) Substituting the expression for the average wage, $w_a = fw_u + (1 - f)w_n$, into the expression for the nonunion wage, $w_n = (1 - bu)w_a/(1 - \beta)$, yields

(1) $$w_n = \frac{(1-bu)}{(1-\beta)}\left[fw_u + (1-f)w_n\right].$$

Substituting the union wage, $w_u = (1 + \mu)w_n$, into equation (1) yields

(2) $$w_n = \frac{(1-bu)}{(1-\beta)}\left[f(1+\mu)w_n + (1-f)w_n\right] = \frac{(1-bu)}{(1-\beta)}\left[(1+\mu f)w_n\right].$$

Simplifying gives us

(3) $(1 - bu)(1 + \mu f) = (1 - \beta)$.

Since $(1 - bu)(1 + \mu f) = 1 + \mu f - bu - b\mu fu$, equation (3) can be rewritten as

(4) $-u(b + b\mu f) = -\beta - \mu f$,

and thus the equilibrium unemployment rate is

(5) $$u = \frac{\beta + \mu f}{b(1+\mu f)}.$$

(b) (i) Substituting $\mu = f = 0.15$, $\beta = 0.06$ and $b = 1$ into equation (5) gives us

(6) $$u = \frac{(0.06) + (0.15)(0.15)}{1 + (0.15)(0.15)} = \frac{0.0825}{1.0225} = 0.081.$$

Equilibrium unemployment is approximately 8.1%, which is higher than the 6% obtained with $\beta = 0.06$ and $b = 1$ in the standard version of this model without a union sector.

In order to determine the proportion by which the cost of effective labor in the union sector exceeds that in the nonunion sector, we need to calculate the equilibrium effort level in each sector. The union wage as a function of the average wage is

(7) $w_u = (1 + \mu)w_n = (1 + \mu)(1 - bu)w_a/(1 - \beta)$.

Substituting equation (7) and the definition of the index of labor-market conditions, $x = (1 - bu)w_a$, into the expression for effort, $e = [(w - x)/x]^{\beta}$, gives us

(8) $$e_u = \left[\frac{\left[(1+\mu)(1-bu)w_a/(1-\beta)\right] - (1-bu)w_a}{(1-bu)w_a}\right]^{\beta} = \left[\frac{(1+\mu)(1-bu) - (1-\beta)(1-bu)}{(1-\beta)(1-bu)}\right]^{\beta},$$

or simply

(9) $$e_u = \left[\left(\frac{1+\mu}{1-\beta}\right) - 1\right]^{\beta} = \left(\frac{\mu+\beta}{1-\beta}\right)^{\beta}.$$

Substituting $w_n = (1 - bu)w_a/(1 - \beta)$ into the expression for effort yields

(10) $$e_n = \left[\frac{\left[(1-bu)w_a/(1-\beta)\right] - (1-bu)w_a}{(1-bu)w_a}\right]^{\beta} = \left[\frac{(1-bu) - (1-\beta)(1-bu)}{(1-\beta)(1-bu)}\right]^{\beta},$$

or simply

(11) $$e_n = \left[\frac{1}{1-\beta} - 1\right]^{\beta} = \left(\frac{\beta}{1-\beta}\right)^{\beta}.$$

In the union sector, it costs a firm w_u to buy one unit of labor that provides e_u units of effective labor. Thus it costs a firm w_u/e_u to buy one unit of effective labor. Using the fact that $w_u = (1 + \mu)w_n$ and equation (9), we can write

$$(12)\quad \frac{w_u}{e_u} = \frac{(1+\mu)w_n}{[(\mu+\beta)/(1-\beta)]^{\beta}}.$$

Similarly, the cost to a nonunion firm of obtaining one unit of effective labor is w_n/e_n. Using equation (11), we can write

$$(13)\quad \frac{w_n}{e_n} = \frac{w_n}{[\beta/(1-\beta)]^{\beta}}.$$

Dividing equation (12) by equation (13) gives us the following ratio of the cost of effective labor in the union to the nonunion sector:

$$(14)\quad \frac{w_u/e_u}{w_n/e_n} = \frac{(1+\mu)w_n}{[(\mu+\beta)/(1-\beta)]^{\beta}}\,\frac{[\beta/(1-\beta)]^{\beta}}{w_n} = (1+\mu)\left(\frac{\beta}{\mu+\beta}\right)^{\beta}.$$

Substituting $\mu = 0.15$ and $\beta = 0.06$ into equation (14) gives us

$$(15)\quad \frac{w_u/e_u}{w_n/e_n} = (1.15)\left(\frac{0.06}{0.21}\right)^{0.06} = 1.0667.$$

Note that although the cost of labor in the union sector exceeds the cost of labor in the nonunion sector by a factor of $(1 + \mu) = 1.15$, the cost of effective labor is only higher by a factor of about 1.07. This is because union workers exert more effort since they are paid a higher wage.

(b) (ii) Substituting $\mu = f = 0.15$, $\beta = 0.03$ and $b = 0.5$ into equation (5) yields

$$(16)\quad u = \frac{(0.03)+(0.15)(0.15)}{0.5[1+(0.15)(0.15)]} = \frac{0.0525}{0.51125} = 0.103.$$

Equilibrium unemployment is now higher at about 10.3%. Substituting $\mu = 0.15$ and $\beta = 0.03$ into equation (14) gives us

$$(17)\quad \frac{w_u/e_u}{w_n/e_n} = (1.15)\left(\frac{0.03}{0.18}\right)^{0.03} = 1.0898.$$

With the elasticity of effort with respect to the wage lower at $\beta = 0.03$ and less weight on unemployment in the index of labor-market conditions, the ratio of the cost of effective labor in the union sector to that in the nonunion sector is now higher.

Problem 9.2

(a) (i) With e fixed at 1 and taking w as given, the firm's problem is to choose L in order to maximize profits as given by

(1) $\pi = L^{\alpha}/\alpha - wL$.

The first-order condition is

(2) $\partial\pi/\partial L = L^{\alpha-1} - w = 0$,

and thus the firm's choice of employment is

(3) $L = w^{-1/(1-\alpha)}$.

Substituting equation (3) into the expression for profits yields

(4) $\pi = w^{-1/(1-\alpha)}/\alpha - w^{[(1-\alpha)-1]/(1-\alpha)} = w^{-\alpha/(1-\alpha)}[(1/\alpha) - 1]$,

and thus the level of profits is

(5) $\pi = [(1 - \alpha)/\alpha]w^{-\alpha/(1-\alpha)}$.

public's estimate of π_2. Equation (18) says that if the variance of the policymaker's taste for inflation, σ_C^2, is very large relative to the variance of the random shocks, σ_ε^2, β will be close to one. The public will attribute most of the above average realization of π_1 to a policymaker with a higher than average c and raise the expectation of π_2 accordingly.

(d) The policymaker knows that her choice of $\hat{\pi}_1$ will affect the public's expectation of inflation in the second period, π_2^e. When π_1 turns out to be high, the public attributes some of this to a policymaker with a high c and accordingly raise π_2^e. From equation (8), we can see that a higher value of π_2^e reduces the expected value of the policymaker's second period objective function. Thus the policymaker chooses a lower $\hat{\pi}_1$ to try and establish a "good reputation" as someone with a low c in order to keep π_2^e down. In the second period, however, there is no future period. Thus there is no need to worry about the effects that this period's inflation will have on future expected inflation.

Problem 10.12

(a) The policymaker chooses inflation in order to maximize her objective function, which is given by $W = c\gamma y - (a\pi^2/2)$, subject to the constraint that output is given by the Lucas Supply function, $y = \bar{y} + b(\pi - \pi^e)$. Thus the policymaker's problem is

(1) $\max_{\pi} \; W = c\gamma[\bar{y} + b(\pi - \pi^e)] - (a\pi^2/2)$.

The first-order condition is

(2) $\partial W/\partial\pi = bc\gamma - a\pi = 0$.

Thus the policymaker's choice of π is

(3) $\pi = bc\gamma/a$.

(b) The public knows the policymaker sets inflation according to equation (3). Thus with rational expectations, expected inflation must equal the expectation of the right-hand side of equation (3):

(4) $\pi^e = E[bc\gamma/a] = bcE[\gamma]/a = bc\bar{\gamma}/a$.

(c) The true social welfare function is given by $W^{SOC} = \gamma y - (a\pi^2/2)$. Taking the expectation of both sides of this expression with respect to the public's information set, so that γ is random, gives us

(5) $E[W^{SOC}] = E[\gamma(\bar{y} + b(\pi - \pi^e)) - (a\pi^2/2)]$,

where we have substituted for $y = \bar{y} + b(\pi - \pi^e)$. Now substitute the policymaker's choice of π, equation (3), and the public's expectation of inflation, equation (4), into equation (5):

(6) $$E[W^{SOC}] = E\left[\gamma\left[\bar{y} + b\left(\frac{bc\gamma}{a} - \frac{bc\bar{\gamma}}{a}\right)\right] - \frac{ab^2c^2\gamma^2}{2a^2}\right].$$

Simplifying yields

(7) $$E[W^{SOC}] = \bar{y}E[\gamma] + \frac{b^2cE[\gamma^2]}{a} - \frac{b^2c\bar{\gamma}E[\gamma]}{a} - \frac{b^2c^2E[\gamma^2]}{2a}.$$

Since $E[\gamma] = \bar{\gamma}$, equation (7) becomes

(8) $$E[W^{SOC}] = \bar{y}\bar{\gamma} + \frac{b^2c}{a}\left[E[\gamma^2] - \bar{\gamma}^2\right] - \frac{b^2c^2E[\gamma^2]}{2a}.$$

Now use the facts that for a random variable X:

(9) $\text{var}(X) = E[X^2] - (E[X])^2$,

and

(10) $E[X^2] = \text{var}(X) + (E[X])^2$.

Here, this means that we can write

(11) $\sigma_\gamma^2 = E[\gamma^2] - \bar{\gamma}^2$,
and
(12) $E[\gamma^2] = \sigma_\gamma^2 + \bar{\gamma}^2$.
Substituting equations (11) and (12) into equation (8) gives us the following expected value of the true social welfare function:

(13) $$E[W^{SOC}] = \bar{y}\bar{\gamma} + \frac{b^2 c}{a}\sigma_\gamma^2 - \frac{b^2 c^2}{2a}\left(\sigma_\gamma^2 + \bar{\gamma}^2\right).$$

(d) To find the first-order condition for the maximization, use equation (13) to set the derivative of the expected value of the social welfare function with respect to c equal to zero:

(14) $$\frac{\partial E[W^{SOC}]}{\partial c} = \frac{b^2}{a}\sigma_\gamma^2 - \frac{b^2 c}{a}\left(\sigma_\gamma^2 + \bar{\gamma}^2\right) = 0.$$

Solving for c yields

(15) $$c = \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \bar{\gamma}^2}.$$

There is a tradeoff here. From equation (3), we can see that choosing a more "conservative" policymaker – one with a low value of c – produces a better performance in terms of average inflation. Such a policymaker would not respond well to the shocks, however. Thus there is some optimal level of "conservatism" that balances these two forces.

The value of c that maximizes the expected value of true social welfare is decreasing in the mean of γ. Since we know that π^e equals π on average (since γ equals $\bar{\gamma}$ on average), output equals full-employment output on average, regardless of the values of c or $\bar{\gamma}$. From equation (3), we can see that if γ is higher on average, inflation will also be higher on average, for a given c. Thus it will be welfare-improving to offset this higher average γ and keep inflation lower on average by having a policymaker with a lower c; that is, having a more "conservative" policymaker.

However, the value of c that maximizes expected social welfare is increasing in the variance of the γ shock. The more variable is the shock, the less "conservative" the central banker should be. Since the policymaker can act after γ is realized, she can choose to offset any deviation in γ from its expected value, which raises welfare. The policymaker will do this only to the extent that she cares about the shock's effect. Thus the more that γ varies, the better it is to have a policymaker who cares about the shock's effect and will act to offset it.

Problem 10.13

(a) Social welfare is higher when the policymaker turns out to be a Type-1, the type that shares the public's preferences concerning output and inflation. The choice of setting $\pi = 0$ in both periods – as the Type-2 policymaker does – is a choice available to the Type-1 policymaker. She chooses not to do this; in order to maximize social welfare, she decides to choose another pair of inflation rates. Since she is attempting to maximize social welfare, welfare must be higher under the choices made by the Type-1 policymaker. For example, as explained in the text, if $\beta < 1/2$, it is optimal for the Type-1 policymaker to choose $\pi_1 = b/a$ and $\pi_2 = b/a$. That must be because it achieves higher welfare than choosing $\pi_1 = 0$, $\pi_2 = 0$.

(b) Expected inflation, π^e, is determined by the public's beliefs. So both the " a' " policymaker and the "a" policymaker face the same π^e, since in either case, the public believes it is facing an " a' "